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MATCHING TO ALL ORDERS AND POWER CORRECTIONS IN HEAVY QUARK EFFECTIVE THEORY

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ABSTRACT

This talk reports on various aspects of the divergence of perturbative expansions in the context of matching QCD onto heavy quark effective theory. Implications for exclusive and inclusive decays of heavy mesons are discussed.

1. Introduction

Heavy quark effective theory (HQET) has become an established tool in the phenomenology of heavy flavours, with applications expanding from their original premises of exclusive heavy flavour transitions into inclusive decays and heavy quark fragmentation.[1] On the theory side, HQET provides a major example for the framework of effective field theories (EFT). Thus, given a Green function with heavy external momenta $p_i = m_Q v + k_i$ and light momenta q_j, the "matching procedure" extracts the heavy mass dependence in the form

$$G_{QCD}(p_i, q_j; m_Q; \alpha) = \sum_l \frac{1}{m_Q^l}\, C_{(l)}\left(\frac{m_Q}{\mu};\alpha\right) G^{(l)}_{HQET}\left(k_i, q_j; \mu; \alpha\right), \tag{1}$$

when the p_i are close to their mass-shell. The validity of Eq. (1) is established inductively in the number of loops – i.e. powers of the coupling α – and to all orders in l, where α and the small off-shellness $|k_i|/m_Q$ are independent parameters. In practice, one is not so much interested in the matching of Green functions as in matrix elements between heavy hadrons. In this case the scale of the off-shellness is provided by the theory itself, $|k_i| \sim \Lambda_{QCD}$. Since $\Lambda_{QCD}/m_Q \sim \exp(1/(2\beta_0\alpha(m_Q)))$, the series in power corrections and the number of loops in the analogue of (1) are organized in terms of a single parameter $\alpha(m_Q)$. Leaving the safe grounds of perturbation theory, one should discuss the presence of power corrections simultaneously with large order, α^n, matching corrections to the coefficient functions $C_{(l)}$. In fact, the series of these corrections diverges as $n \to \infty$ and one source of divergence originates from low momentum regions, which one would like to factor into nonperturbative parameters that appear in power

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corrections. The summation of the divergent series introduces ambiguities of the same order of magnitude as these nonperturbative parameters, which must therefore also be ambiguous. This divergence pattern – known as renormalons – and its consequences have a long history[2] in the context of the short-distance expansion and QCD sum rules. In this talk I discuss the renormalon phenomenon in HQET and its (ir)relevance for phenomenology.[3,4]

2. Renormalon Structure of HQET

Investigations of large orders in perturbation theory naturally have to resort to some kind of approximation. Since renormalons are associated with the integration over logarithms provided by vacuum polarizations, some insight can be obtained from the restriction to the class of diagrams generated from insertion of a chain of fermion bubbles into the low order diagrams. Taking Borel transforms and defining $u = -\beta_0 t$, factorial divergence of perturbative series in α translates into singularities of their Borel transforms in u. Singularities at positive u imply non-Borel summability and an ambiguity in the definition of the sum of the original divergent series. In the following, $\overline{MS}$ renormalization in QCD and HQET will be assumed, though m_Q need – and should – not coincide with the renormalized mass m of the heavy quark.

The general structure of the Borel transformed version of Eq. (1) can be described as follows: The Green functions $G^{(l)}_{HQET}$ in HQET (with operator insertions) are power-like divergent. Explicit power divergences are absent in dimensional regularization, but they do not disappear without a trace in $\overline{MS}$. Subtractions are such that they leave divergent series expansions with non-summable ultraviolet (UV) renormalons at positive half-integer u. It is natural to associate this divergence with integration over large internal momenta, $k \gg \mu$, though not straightforward, because integrals are defined by analytic continuation. The coefficient functions, $C_{(l)}$, have singularities at positive half-integers, too, which stem from small internal momenta, $k \ll \mu$. Infrared (IR) renormalons in coefficient functions cancel with the UV renormalons – up to the singularities present already on the l.h.s. of Eq. (1). This cancellation takes place over different orders in the expansion in $1/m_Q$. Thus, if this expansion is truncated at a certain order, summation of the perturbative series produces an ambiguous result, which is removed only by including higher orders in $1/m_Q$.

As an illustration, consider the matching of the inverse propagator of a heavy quark within the above approximation. The first two terms of the heavy quark expansion are given by

$$S^{-1}(p,m;u) = m_Q\left(\frac{m}{\mu};u\right) - m_{pole}\left(\frac{m}{\mu};u\right) + C\left(\frac{m_Q}{\mu};u\right)\star\left(vk\delta(u) - \Sigma_{eff}(vk;u)\right) + \ldots \tag{2}$$

The explicit expressions for the ingredients of Eq. (2) lead to the conclusions[3]:

(a) The pole mass of the heavy quark in the first term on the r.h.s. has an IR renormalon[5] at $u = 1/2$ when expressed in terms of m. Thus, the pole mass can not be defined to an accuracy better then Λ_{QCD}. While this might be expected, the interesting point is that perturbation theory itself indicates its failure through its divergence. As a consequence, when the heavy quark expansion is applied to hadronic parameters, the

quantity $\Lambda_{H_Q} \equiv m_{H_Q} - m_{pole}$, defined as the difference between the heavy hadron mass and the pole mass of the quark in the heavy quark limit, contains an ambiguity of order Λ_{QCD}. Note, however, that this ambiguity, though of the same order of magnitude as Λ_{H_Q} itself, is not related to bound state effects contained in Λ_{H_Q}, but can be traced to the long range part of the Coulomb field of the quark. Thus, the effect is universal and obviously cancels in mass differences.

(b) Off mass-shell, the l.h.s. of Eq. (2) is non-singular at $u = 1/2$. As anticipated from the general discussion, for $k \neq 0$ the IR renormalon at $u = 1/2$ in the pole mass cancels exactly against an UV renormalon at this position in the self-energy of the static quark, Σ_{eff}, computed from the leading term in the HQET Lagrangian, $\bar{h}_v v \cdot D h_v$. This UV renormalon arises, since, in contrast with full QCD, the self-energy of the static quark is linearly UV divergent. This is nothing but the linear divergence of a static point charge known from classical electrodynamics, which reappears in HQET, where the quark mass is considered larger than the UV cutoff.

(c) To reproduce the r.h.s. of Eq. (2) from HQET without a residual mass term, the first term must vanish and the expansion parameter m_Q has to coincide with the pole mass. This destroys artifically the cancellation of renormalon poles, part of which become hidden in the expansion parameter, which then is not defined beyond perturbation theory. From this point of view it is conceptually advantageous to use the freedom to add a small residual mass term $-\delta m \bar{h}_v h_v$ to the effective Lagrangian, such that both Σ_{eff} computed from HQET with residual mass and the expansion parameter $m_Q = m_{pole} - \delta m$ are formally free from an ambiguity due to a renormalon at $u = 1/2$. This can be accomplished either by $\delta m \propto \mu \sum c_n \alpha(\mu)^n$ ($\mu \ll m_Q$) with c_n adjusted to the UV renormalon divergence or a formally ambiguous $\delta m \propto \mu \exp(1/(2\beta_0 \alpha(\mu)))$, adjusted to compensate the ambiguities of the Borel sums.

3. Implications

Exclusive heavy flavour decays are governed by matrix elements of the weak current between heavy hadron states. HQET is particularly effective in restricting the number of independent form factors in the infinite mass limit and parameterizing the corrections to this limit. These corrections involve new nonperturbative form factors and typically the ratio Λ_{H_Q}/m_Q, which controls the size of these corrections. Since physical quantities must be unambiguous, the ambiguity in the definition of Λ_{H_Q} implies that the matching corrections in leading order diverge (with an IR renormalon at $u = 1/2$), such that the ambiguity of their sum compensates the ambiguity in Λ_{H_Q}, which has been inferred from the pole mass. It is easy to see that an IR renormalon at $u = 1/2$ will indeed occur. The leading order matching corrections are conveniently calculated by comparing the current insertions between on-shell quark states in the full and the effective theory. In the IR, the integrals behave like d^4k/k^4, but the coefficient is the same in the full and the effective theory and the logarithmic IR divergence cancels as it must be. The next term in the expansion for small k, d^4k/k^3, is different, however. Although this region gives a small and finite contribution to the coefficient function in first order, it is greatly amplified by large powers of logarithms, $\ln^n k^2/\mu^2$, in higher orders, which produces the required divergence of the series.

Thus, the structure of the heavy quark expansion is conceptually quite similar

to the short distance expansion.[2] Power corrections must be added with care, since the summation of perturbative corrections, which is never performed in practice, can produce effects of the same order. In the particular case of Λ_{H_Q} the situation might be more favorable phenomenologically. This parameter contains the effect of the light spectators in the heavy hadron, which appears first at this order and which is clearly not related to renormalon ambiguities. Given the large value $\Lambda_P \approx 500\,\mathrm{MeV}$, favored for pseudoscalar mesons, one may argue that it is dominated by the spectator and renormalon effects may be neglected in comparison.

EFT calculations are most conveniently done in $\overline{MS}$. Since loop integrations run unrestricted over all momenta, renormalons inevitably appear in the matching corrections. Alternatively, one might imagine cutting the low momentum region explicitly from the Feynman integrals, absorbing them into nonperturbative parameters in higher orders of the $1/m_Q$-expansion. Disregarding the practical difficulties of this procedure, there is a definite drawback: The nonperturbative parameters are no longer universal and therefore useless (beyond a certain accuracy). However, strictly within the framework of EFT, where nonperturbative effects are not calculated but parameterized, the renormalon phenomenon never constitutes a difficulty. Indeed, if one accepts the assumption that IR renormalons in the coefficient functions are in one-to-one correspondence with (ambiguities of) nonperturbative parameters, one may eliminate these parameters up to a certain order in $1/m_Q$ in favor of physical quantites to obtain predictions for other physical quantities entirely in terms of physical quantities (up to a certain order in $1/m_Q$). Then, the relation between measurable quantities will always be free from renormalons up to renormalons corresponding to a still higher order in $1/m_Q$. (Depending on the definition of the coupling, it might be necessary to eliminate the coupling as well.)

The significance of renormalons appears in two respects: First, when one attempts to calculate the subleading nonperturbative parameters such as Λ_{H_Q}, e.g. from QCD sum rules or lattice gauge theory. In the latter case, the difficulty is rather profound and appears as explicit power divergences in the lattice spacing that require "nonperturbative subtractions".[6] Second, the structure of renormalons serves as a check that IR effects are indeed correctly parameterized by matrix elements of higher dimensional operators. As an example, consider the semileptonic decay width for a B meson. To leading order in $1/m_b$, the width is naturally proprotional to $G_F^2 m_{b,pole}^5$. Operator product expansion and HQET predict corrections to the free quark decay starting[7] from $1/m_b^2$ in apparent conflict with an ambiguity of order Λ_{QCD} from the IR region in the pole mass. In this case it turns out that the renormalon in the radiative corrections to the free quark decay cancels exactly against the one hidden in the pole mass, when the pole mass is eliminated in favor of a mass parameter that is not sensitive to the Coulomb tail of the self-energy,[4,5] implying consistency with the $1/m_b$-expansion.

References

1. A (selective) choice of reviews is, B. Grinstein, *Annu. Rev. Nucl. Part. Sci.* **42** (1992) 101; M. Neubert, SLAC–PUB–6263, to appear in Phys. Rep.; M. Shifman, TPI-MINN-94/17-T
2. F. David, *Nucl. Phys.* **B234** (1984) 237

3. M. Beneke and V. M. Braun, MPI-PhT/94-9 [hep-ph/9402364], to appear in Nucl. Phys. B
4. M. Beneke, V. M. Braun and V. I. Zakharov, MPI-PhT/94-18, May 1994 [hep-ph/9405304]
5. This observation is also contained in I. I. Bigi *et al.*, TPI-MINN-94/4-T
6. L. Maiani, G. Martinelli and C. T. Sachrajda, *Nucl. Phys.* **B368** (1992) 281
7. J. Chay, H. Georgi and B. Grinstein, *Phys. Lett.* **B247** (1990) 399; I. Bigi, N. Uraltsev and A. Vainshtein, *Phys. Lett.* **B293** 430; (E) **B297** (1993) 477